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**Curriculum outline**

Core candidates only study the Core syllabus while Extended candidates will need to study both the Core and the Extended topic *(blue text)*.

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* Topic 19 Straight line graphs becomes a Core topic for the first examination in 2006.
** The italicised areas of study are not examined until the first examination in 2006.
Number, set notation and language

Use natural numbers, integers, prime numbers, common factors and multiples, rational and irrational numbers, real numbers, number sequences, recognise patterns in sequences, generalise to simple algebraic statements (i.e. nth term)

Natural numbers
The natural numbers are the counting numbers, i.e. 1, 2, 3, 4, etc.

Integers
Integers are whole numbers; they can be positive or negative, e.g. −5, 3, 25.

If the number contains a fraction part or a decimal point, then it cannot be an integer. (For example, the numbers 4.2 and 1\frac{1}{2} are not integers.

Prime numbers
Numbers that can only be divided by themselves and one, e.g. 2, 3, 5, 7, 11, 13, are prime numbers.

Note that 1 is not considered prime and 2 is the only even prime number.

Factors
A number is a factor of another number if it divides exactly into that number without leaving a remainder, e.g.

the factors of 6 are 1, 2, 3, 6
the factors of 15 are 1, 3, 5, 15

To find the factors of a number quickly, find which numbers were multiplied together to give that number.

For example, the products which give 8 are 1 \times 8 or 2 \times 4
so the factors of 8 are 1, 2, 4, 8

Highest Common Factor (HCF)
As the name suggests, this is simply the highest factor which is common to a group of numbers.

Example 1
Find the HCF of the numbers 6, 8 and 12.
Factors of 6 are 1, 2, 3, 6
Factors of 8 are 1, 2, 4, 8
Factors of 12 are 1, 2, 3, 4, 6, 12
As the number 2 is a the highest factor of the three numbers, \text{HCF} = 2

Multiples
Multiples means ‘the times table’ of a number, e.g. multiples of 4 are 4, 8, 12, 16, \ldots
multiples of 9 are 9, 18, 27, 36, \ldots
Lowest Common Multiple (LCM)
This is the lowest multiple which is common to a group of numbers. It is found by listing all the
multiples of a group of numbers and finding the lowest number which is common to each set of
multiples.

Example 2
Find the LCM of the numbers 2, 3, and 9.
Multiples of 2 are 2, 4, 6, 8, 10, 12, 14, 16, 18, 20 . . .
Multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24 . . .
Multiples of 9 are 9, 18, 27, 36 . . .
The number 18 is the lowest number that occurs as a multiple of each of the numbers.
So the LCM is 18.

Rational numbers
Rational numbers are numbers which can be shown as fractions; they either terminate or have
repeating digits, e.g. \(\frac{3}{4}\), 4.333, 5.34 34 34, . . . etc.

Note that recurring decimals are rational.

Irrational numbers
An irrational number cannot be shown as a fraction, e.g. \(\sqrt{2}\), \(\sqrt{3}\), \(\sqrt{5}\), \(\pi\). Since these numbers
never terminate, we cannot possibly show them as fractions.

The square root of any odd number apart from the square numbers is irrational. (Try them on your
calculator, you will find that they do not terminate.) Also, any decimal number which neither
repeats nor terminates is irrational.

For more information on square numbers see Special number sequences on page 4.

Number sequences
A number sequence is a set of numbers that follow a certain pattern, e.g.

1, 3, 5, 7, . . . Here the pattern is either consecutive odd numbers or add 2.
1, 3, 9, 27, . . . The pattern is \(3 \times\) previous number.

The pattern could be add, subtract, multiply or divide. To make it easier to find the pattern,
remember that for a number to get bigger, generally you have to use the add or multiply operation.
If the number gets smaller, then it will usually be the subtract or divide operation.

Sometimes the pattern uses more than one operation, e.g.

1, 3, 7, 15, 31, . . . Here the pattern is multiply the previous number by 2 and then add 1.

The \(n\)th term
For certain number sequences it is necessary, and indeed very useful, to find a general formula for
the number sequence.

Consider the number sequence 4, 7, 10, 13, 16, 19. We can see that the pattern is add 3 to the
previous number, but what if we wanted the 50th number in the sequence?

This would mean continuing the sequence up to the 50th value, which is very time-consuming and
indeed unnecessary.

A quicker method is to find a general formula for any value of \(n\) and then substitute 50 to find its
corresponding value. These examples show the steps involved.
Example 3

Find the \(n\)th term and hence the 50th term of the following number sequence:

\[
\begin{align*}
4 & \quad 7 & \quad 10 & \quad 13 & \quad 16 & \quad 19 & \ldots
\end{align*}
\]

We can see that you add 3 to the previous number. To find a formula, follow the steps below.

**Step 1**
Construct a table and include a difference row.

<table>
<thead>
<tr>
<th>(n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence</td>
<td>4</td>
<td>7</td>
<td>10</td>
<td>13</td>
<td>16</td>
<td>19</td>
</tr>
</tbody>
</table>

| 1st difference | 3 | 3 | 3 | 3 | 3 |

**Step 2**
Look at the table to see where the differences remain constant.

We can see that the differences are always 3 in the first difference row. This means that the formula involves \(3n\). If we then add 1, we get the original terms in the sequence:

When \(n = 1\), \[
3 \times (1) + 1 = 4
\]
When \(n = 2\), \[
3 \times (2) + 1 = 7
\]

**Step 3**
Form a general \(n\)th term formula and check.

Knowing that we have to multiply \(n\) by 3 and then add 1:

\[\text{\(n\)th term} = 3n + 1\]

This formula is extremely powerful as we can now find the corresponding term in the sequence for any value of \(n\). To find the 50th term in the sequence:

Using \(n\)th term = \(3n + 1\) when \(n = 50\),

\[3 \times (50) + 1 = 151\]

Therefore the 50th term in the sequence will be 151.

This is a much quicker method than extending the sequence up to \(n = 50\).

Sometimes, however, we have sequences where the first difference row is not constant, so we have to continue the difference rows as shown in Example 4.

Example 4

Find the \(n\)th term and hence the 50th term for the following sequence:

\[
\begin{align*}
0 & \quad 3 & \quad 8 & \quad 15 & \quad 24 & \quad 35 & \ldots
\end{align*}
\]

Construct a table.

<table>
<thead>
<tr>
<th>(n)</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sequence</td>
<td>0</td>
<td>3</td>
<td>8</td>
<td>15</td>
<td>24</td>
<td>35</td>
</tr>
</tbody>
</table>

| 1st difference | 3 | 5 | 7 | 9 | 11 |

| 2nd difference | 2 | 2 | 2 | 2 |

Now we notice that the differences are equal in the second row, so the formula involves \(n^2\). If we square the first few terms of \(n\) we get: 1, 4, 9, 16, etc. We can see that we have to subtract 1 from these numbers to get the terms in the sequence. So

\[\text{\(n\)th term} = n^2 - 1\]

Now we have the \(n\)th term, to find the 50th term we use simple substitution:

\[(50)^2 - 1 = 2499\]

Note that some more complicated sequences will require a third difference row (\(n^3\)) for the differences to be constant, so we have to manipulate \(n^3\) to get the final formula.
### Special number sequences

**Square numbers**

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<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
<td>9</td>
<td>16</td>
<td>25</td>
<td>...</td>
</tr>
<tr>
<td>(1^2)</td>
<td>(2^2)</td>
<td>(3^2)</td>
<td>(4^2)</td>
<td>(5^2)</td>
<td></td>
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</tbody>
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The counting numbers squared.

**Cubed numbers**

<p>| | | | | | |</p>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>8</td>
<td>27</td>
<td>64</td>
<td>125</td>
<td>...</td>
</tr>
<tr>
<td>(1^3)</td>
<td>(2^3)</td>
<td>(3^3)</td>
<td>(4^3)</td>
<td>(5^3)</td>
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The counting numbers cubed.

**Triangular numbers**

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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3</td>
<td>6</td>
<td>10</td>
<td>15</td>
<td>...</td>
</tr>
</tbody>
</table>

Each number can be shown as a triangle, or simply add an extra number each time.

![Diagram of triangular numbers](image-url)
Definition of a set
A set is a collection of objects, numbers, ideas, etc. The different objects, numbers, ideas, etc. are called the elements or members of the set.

Example 1
Set A contains the even numbers from 1 to 10 inclusive. Write this as a set.
The elements of this set will be 2, 4, 6, 8, 10, so we write:
\[ A = \{2, 4, 6, 8, 10\} \]

Example 2
Set B contains the prime numbers between 10 and 20 inclusive. Write this as a set.
The elements of this set will be 11, 13, 17 and 19. So
\[ B = \{11, 13, 17, 19\} \]

\( n(A) \), number of elements in set A
We count the number of elements in the set.

Example 3
If set C contains the odd numbers from 1 to 10 inclusive, find \( n(C) \).
\[ C = \{1, 3, 5, 7, 9\} \]
There are 5 elements in the set. So
\[ n(C) = 5 \]

\( \in \), is an element of, and \( \notin \), not an element of
The symbols \( \in \) and \( \notin \) indicate whether or not a certain number is an element of the set.

Example 4
Set A = \{2, 5, 6, 9\}. Describe which of the numbers 2, 3 or 4 are elements and which are not elements of set A.
Set A contains the element 2, therefore 2 \( \in \) A.
Set A does not contain the elements 3 or 4, therefore 3, 4 \( \notin \) A.

\( \xi \), the universal set, and \( A' \), the complement of a set
The universal set, \( \xi \), for any problem is the set which contains all the available elements for that problem. The complement of a set A is the set of elements of \( \xi \) which do not belong to A.

Example 5
The universal set is all of the odd numbers up to and including 11. List the universal set.
\[ \xi = \{1, 3, 5, 7, 9, 11\} \]

Example 6
The complement of a set A is the set of elements of \( \xi \) in Example 5 which do not belong to A. List the complement of set A.
If \( A = \{3, 5\} \) and \( \xi = \{1, 3, 5, 7, 9, 11\} \), then \( A' = \{1, 7, 9, 11\} \).
The empty set \( \emptyset \) or \( \{ \} \)

The empty set contains no elements, e.g. for some pupils the set of people who wear glasses in their family will have no members.

Note that this is sometimes referred to as the null set.

Subsets \( A \subseteq B \)

If all the elements of a set \( A \) are also elements of a set \( B \) then \( A \) is said to be a subset of \( B \). Every set has at least two subsets, itself and the null set.

Example 7

List all the subsets of \( \{a, b, c\} \).

The subsets are \( \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\} \) and \( \{a, b, c\} \), because all of these elements can occur in their own right inside the main set.

Note that the number of subsets can be found by using the formula \( 2^n \), where \( n \) = number of elements in the set.

Intersection \( A \cap B \) and union \( A \cup B \)

The intersection of two sets \( A \) and \( B \) is the set of elements which are common to both \( A \) and \( B \). Intersection is denoted by the symbol \( \cap \).

The union of the sets \( A \) and \( B \) is the set of all the elements contained in \( A \) and \( B \). Union is denoted by the symbol \( \cup \).

Example 8

If \( A = \{2, 3, 5, 8, 9\} \) and \( B = \{1, 3, 4, 8\} \) find:

(a) \( A \cap B = \{3, 8\} \) These elements are common to both sets.

(b) \( A \cup B = \{1, 2, 3, 4, 5, 8, 9\} \) These are the total elements contained in both \( A \) and \( B \).

Venn diagrams

Set problems may be solved by using Venn diagrams. The universal set is represented by a rectangle and subsets of this set are represented by circles or loops. Some of the definitions explained earlier can be shown using these diagrams as follows:
Obviously there are many different arrangements possible with these diagrams but now let us try some more difficult problems.

**Example 9**

A = \{3, 4, 5, 6\}, B = \{2, 3, 5, 7, 9\} and \(\xi = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11\}\). Draw a Venn diagram to represent this information. Hence write down the elements of:

(a) \(A'\)  
(b) \(A \cap B\)  
(c) \(A \cup B\)

We only have two sets (A, B), so there are two circles inside the universal set:

(a) \(A' = \{1, 2, 7, 8, 9, 10, 11\}\)  
(b) \(A \cap B = \{3, 5\}\)  
(c) \(A \cup B = \{2, 3, 4, 5, 6, 7, 9\}\)

---

**Problems with the number of elements of a set**

For two intersecting sets A and B we can use the rule:

\[
\text{n}(A \cup B) = \text{n}(A) + \text{n}(B) - \text{n}(A \cap B)
\]

This example shows how to use this rule.

**Example 10**

In a class of 25 members, 15 take history, 17 take geography and 3 take neither subject. How many class members take both subjects?

Let \(H = \) set of History students, so \(n(H) = 15\). Let \(G = \) set of Geography students, so \(n(G) = 17\) and \(n(H \cup G) = 22\) (since 3 students take neither subject). Let \(x\) represent the number taking both subjects.

Now we can draw the Venn diagram.

Using the formula:

\[
\text{n}(H \cup G) = \text{n}(H) + \text{n}(G) - \text{n}(H \cap G)
\]

\[
22 = 15 + 17 - x
\]

\[
x = 10
\]

Hence the number taking both subjects is 10.

The History only region is labelled \(15 - x\). This is because a number of these students take Geography also.

Note that this formula can be used for any problem involving two sets.